An Inquisitive Approach to Interrogative Inquiry

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Interrogative Model of Inquiry Workshop

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Goals and Motivations

Goals

- To investigate **interrogative inquiry** in **conversations**.
  - Socratic dialogues,
  - Rational agency.
- To build on **inquisitive semantics** and **pragmatics**.

$\Rightarrow$ Investigation of the Language-Game of Interrogative Inquiry.

Motivations

- **Inquisitive semantics** offers:
  - A representation of embedded questions,
  - A semantic categorization of questions and assertions,
  - A precise notion of complete and partial answerhood.
- **Inquisitive pragmatics** offers:
  - An account of the behavior of questions and answers in conversations.

Outline

1. The Inquisitive Modelling of Questions and Answers
2. Interrogative Rule
3. Interrogative Protocol, Inquiry and Consequence
4. Logical Aspects
5. Computational Aspects
6. Strategic Aspects of Inquiry: The Algorithmic View

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Inquisitive Semantics: Basic Notions

Definition (Language $\mathcal{L}$)

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \varphi \to \psi \quad \textit{with } p \in \mathcal{P}$$

Definition (index and state)

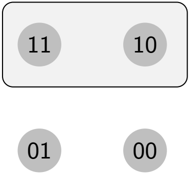
- *An index v is a binary valuation $v : \mathcal{P} \to \{0,1\}$,*
- *A state is a non-empty set of indices.*

Definition (Support)

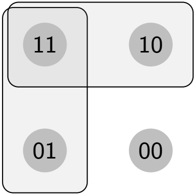
- $s \models p$ *iff* $\forall v \in s : v(p) = 1$
- $s \models \neg\varphi$ *iff* $\forall t \subseteq s :$ *not* $t \models \varphi$
- $s \models \varphi \vee \psi$ *iff* $s \models \varphi$ *or* $s \models \psi$
- $s \models \varphi \wedge \psi$ *iff* $s \models \varphi$ *and* $s \models \psi$
- $s \models \varphi \to \psi$ *iff* $\forall t \subseteq s$: *if* $t \models \varphi$ *then* $t \models \psi$

Definition (Possibility, Proposition and Truth Set)

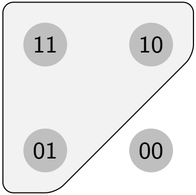
- *A possibility for φ in s is a max. substate of s supporting φ.*
- *The proposition expressed by φ in s, denoted by $s[\varphi]$, is the set of possibilities for φ in s.*
- *The truth set of φ in s, denoted by $s|\varphi|$, is the set of indices in s where φ is classically true.*



Possibility p



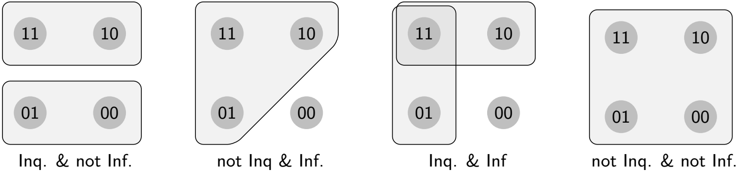
Proposition $p \vee q$



Truth-set $|p \vee q|$

Definition (Informativeness and Inquisitiveness)

- *φ is inquisitive in s iff $s[\varphi]$ contains at least two possibilities.*
- *φ is informative in s iff $s[\varphi]$ contains at least one possibility and $\bigcup s[\varphi] \subset s$.*



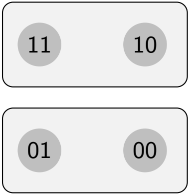
Inquisitive Semantics: Question and Assertion

Definition (Question and Assertion)

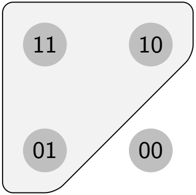
- *φ is a question in s iff φ is inquisitive and not informative in s.*
- *φ is an assertion in s iff φ is not inquisitive and informative in s.*

Definition (Settledness)

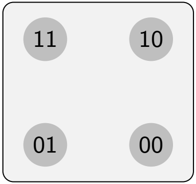
- *We say that φ is settled in s iff $s[\varphi] = \{s\}$.*



Question



Assertion

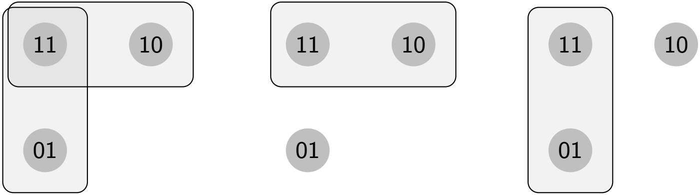


Settled

Inquisitive Semantics: Answerhood

Definition (Answerhood)

- φ *is an* answer *to* ψ *in* s *iff* $s|\varphi|$ *coincides with the union of a set of possibilities for* ψ *in* s *and* φ *is informative in* s.
- φ *is a* complete answer *to* ψ *in* s *iff* $s|\varphi|$ *coincides with one of the possibilities for* ψ *in* s.
- φ *is a* partial answer *to* ψ *in* s *iff* φ *is an answer but not a complete answer to* ψ *in* s.



Question $p \vee q$ | Answer p | Answer q

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The Notion of Interrogative Rule

When we are recording the successive steps of interrogative inquiry on paper, logical inference steps and interrogative steps look rather similar. The former are steps from a premise to a conclusion; the latter are steps from the presupposition(s) of a question to its answer.
[Hintikka, Socratic Epistemology, p. 71]

Splitting the Interrogative Rule into Two Components

A pragmatic rule for **answering**: which governs the **production of answers** to questions given (i) the informational state of the answerer and (ii) the common ground of the conversation.

A pragmatic rule for **updating**: which governs the way the conversation, i.e., the common ground and the informational states of the participants, is **updated** after the reception of an answer to a question.

The Language-Game of Interrogative Inquiry

Basic Rules

- We designate one of the participants as the **inquirer** and the other participants as the **oracles**,
- Each interrogative step takes the form of a **question asked by the inquirer** and (eventually) answered by the oracles or the inquirer,
- Each question asked by the inquirer is **directed** towards a particular conversational participant.

Definition (Conversational state)

A **conversational state** C *is a* $\mathcal{S}$*-tuple* $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$ *s.t.:*

- σ *denotes the common ground of the conversation,*
- τ_I *denotes the informational state of the inquirer,*
- $\tau_{O_1}, \ldots, \tau_{O_n}$ *denote the informational states of the oracles,*

such that: **(1)** $\tau_I, \tau_{O_1}, \ldots, \tau_{O_n} \subseteq \sigma$ *and* **(2)** $\left(\bigcap_{1 \leq i \leq n} \tau_{O_i}\right) \cap \tau_I \neq \emptyset$.

Conversational State

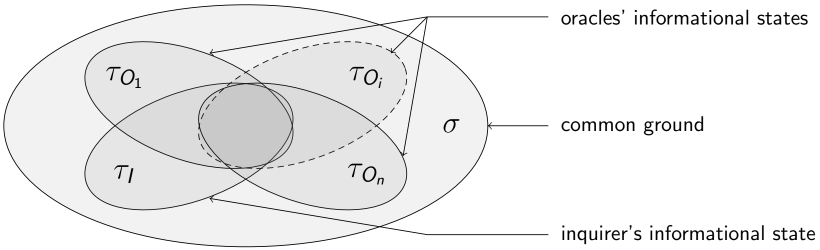

Figure: A conversational state $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$.

Pragmatic Rule(s) for Answering

Definition (Answer)

Let $\varphi, \psi \in \mathcal{L}$ *and* $\sigma, \tau \in \mathcal{S}$ *such that* $\tau \subseteq \sigma$, ψ *is a question and* φ *an assertion in* σ. *We say that* φ *is an answer to* ψ *for* τ *in* σ *if:*

- φ *is an answer to* ψ *in* σ,
- $\tau|\varphi| = \tau$.

$$\varphi \in \textbf{Answers}(\psi, \tau, \sigma) \Leftrightarrow \varphi \text{ is an answer to } \psi \text{ for } \tau \text{ in } \sigma$$

Definition (Answering rule)

$$\begin{array}{cccc} A : & \psi, \tau, \sigma & \longmapsto & A(\psi, \tau, \sigma) \\ & \mathcal{L} \times \mathcal{S} \times \mathcal{S} & \longrightarrow & \mathcal{L} \end{array}$$

with $A(\psi, \tau, \sigma)$ *defined for all* (ψ, τ, σ) *s.t.* ψ *is a question in* σ *by*

$$A(\psi, \tau, \sigma) = \begin{cases} \varphi \in \textbf{Answers}(\psi, \tau, \sigma) & \text{if } \textbf{Answers}(\psi, \tau, \sigma) \neq \emptyset, \\ \top & \text{otherwise.} \end{cases}$$

Pragmatic Rule for Updating

Definition (Updating rule)

The updating rule is a partial function

$$\begin{array}{cccccc} U & : & C, \varphi & \longmapsto & C|\varphi \\ & & \mathcal{C} \times \mathcal{L} & \longrightarrow & \mathcal{C} \end{array}$$

where $C|\varphi$ *is defined for all* (C, φ) *such that* $s|\varphi| \neq \emptyset$*, with*

- $s := \left(\bigcap_{1 \leq i \leq n} \tau_{O_i}\right) \cap \tau_I$,
- $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$,

by

$$C|\varphi = (\sigma|\varphi, \tau_I|\varphi, \tau_{O_1}|\varphi, \ldots, \tau_{O_n}|\varphi),$$

with $t|\varphi = t|\varphi|$ *for* $t \in \mathcal{S}$.

Interrogative Rule

Definition (Interrogative rule)

- *Let $n \in \mathbb{N}$ representing the number of oracles,*
- *Let A be an answering rule.*

The interrogative rule associated to A and n is a partial function

$$\begin{array}{rccc} I_n & : & C, \psi, i & \longmapsto & C|^?_i \psi \\ & & \mathcal{C}^n \times \mathcal{L} \times [\![0, n]\!] & \longrightarrow & \mathcal{C} \end{array}$$

where $C|^?_i \psi$ is defined for all (C, ψ, i) s.t. ψ is a question in σ by

$$C|^?_i \psi = C|A(\psi, \tau_i, \sigma) = U(C, A(\psi, \tau_i, \sigma)).$$

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Interrogative Protocol

Definition (Interrogative protocol)

- *Let $n \in \mathbb{N}$ be the number of oracles,*
- *Let $C \in \mathcal{C}^n$ be the starting conversational state,*
- *Let I_n be an interrogative rule.*

The interrogative protocol $P_?(C, I_n)$ based on C and I_n is defined as a tree built as follows:

Root: *the root of the tree is C,*

Expanding rule: *if $C' = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$ is a node of the tree, then*

- *for each formula φ s.t. φ is a question in σ,*
- *For each $i \in [\![0, n]\!]$,*

$\Rightarrow$ *C' has a successor $C'|^?_i \varphi = I_n(C', \varphi, i)$.*

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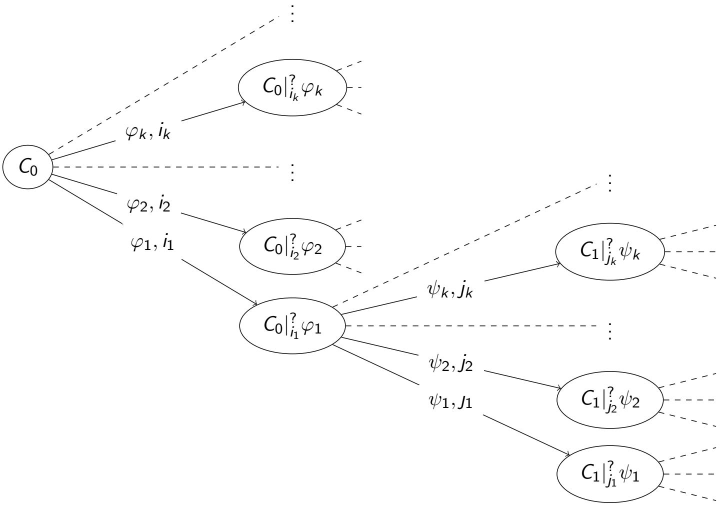

Definition (Interrogative inquiry)

Let $P_?(C, I_n)$ be an interrogative protocol.

An interrogative inquiry in $P_?(C, I_n)$ is a **finite sequence**

$$\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$$

of elements in $\mathcal{L} \times [\![0, n]\!]$ which corresponds to the **labels** *of a* **finite branch** *in $P_?(C, I_n)$ from the root C.*

This definition fits:

- The intuitive representation of interrogative inquiries as **sequences** of **directed questions**.
- The intuitive idea that interrogative inquiries take place in a particular **temporal process** governs by certain rules.

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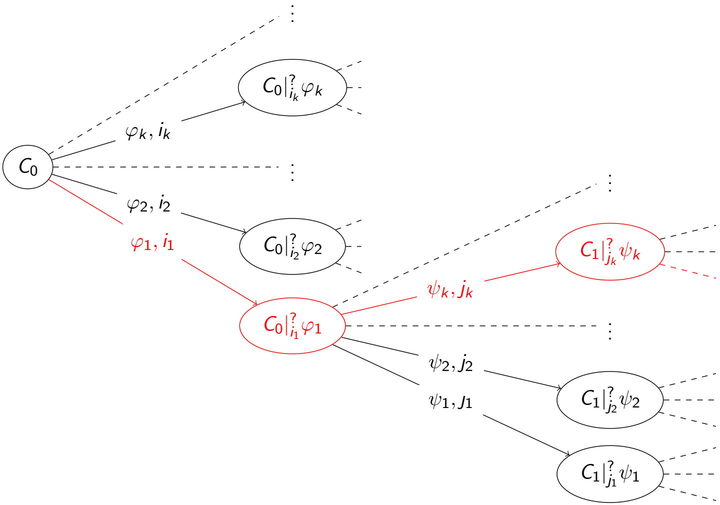

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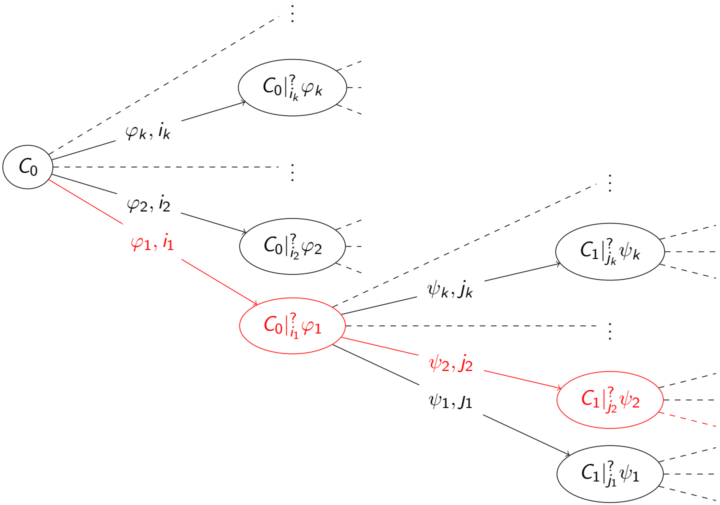

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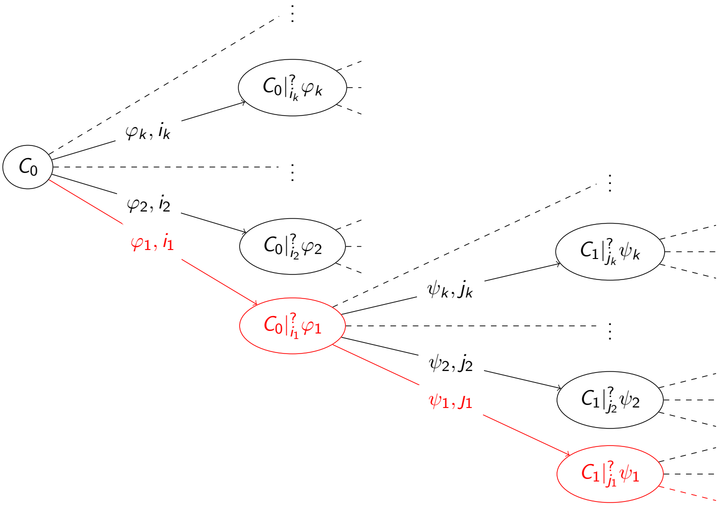

Interrogative Consequence

The Intuitive Idea

φ is an interrogative consequence in $P_?(C, I_n)$ if there exists an interrogative inquiry in $P_?(C, I_n)$ leading to a conversational state in which φ has been established in the common ground.

'Established': Inquisitive and Classical Views

- We say that φ has been classically established in the common ground σ when $\sigma|\varphi| = \sigma$,
- We say that φ has been inquisitively established in the common ground σ when $\sigma[\varphi] = \{\sigma\}$.

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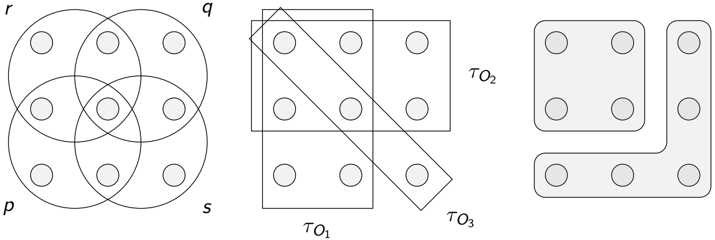
Illustrative Example: Stating the Ground



Common ground σ. Informational states. Inquiry goal $r?$.

Figure: Conversation state $C = (\sigma, \tau_I, \tau_{O_1}, \tau_{O_2}, \tau_{O_3})$ and inquiry goal $r?$.

Inquiry 1	**Inquiry 2**	**Inquiry 3**
$O_3 : \; r?$	$O_2 : \; s \rightarrow q?$	$O_2 : \; p \wedge \neg q \rightarrow r?$
$O_1 : \; p \vee q \vee r \vee s$	$O_3 : \; p \rightarrow s?$	$O_1 : \; p?$
$O_2 : \; p \vee r$	$O_1 : \; q \rightarrow r?$	$O_3 : \; \neg q?$

Table: Three examples of interrogative inquiries in $P_?(C, I_n)$.

Illustrative Example: Inquiry 1

Figure: Interrogative inquiry 1.

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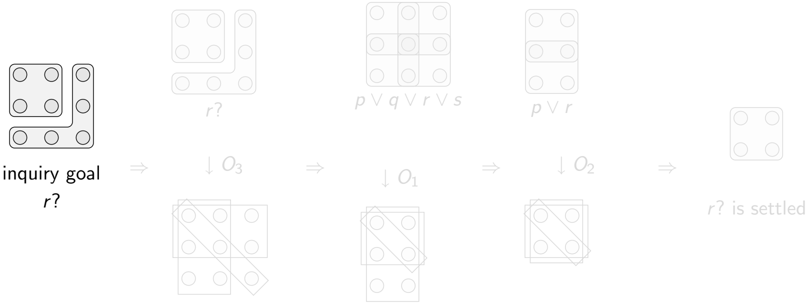

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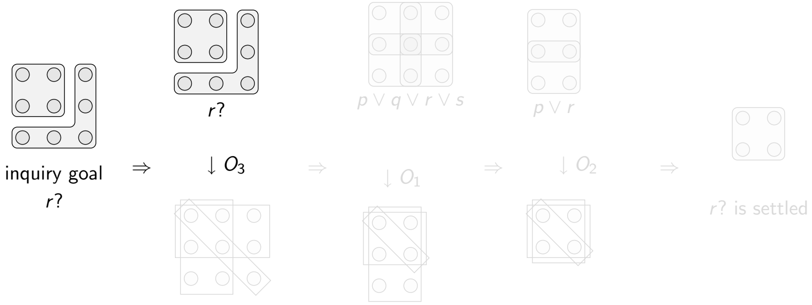

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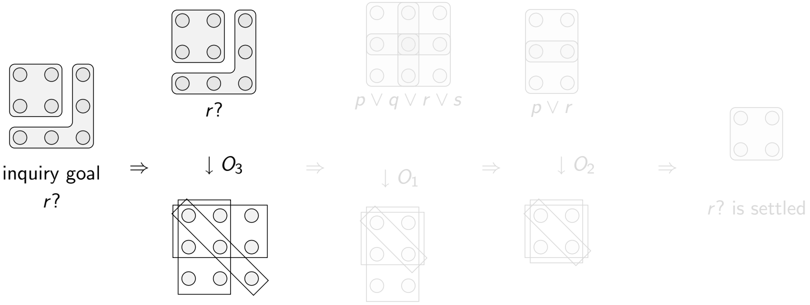

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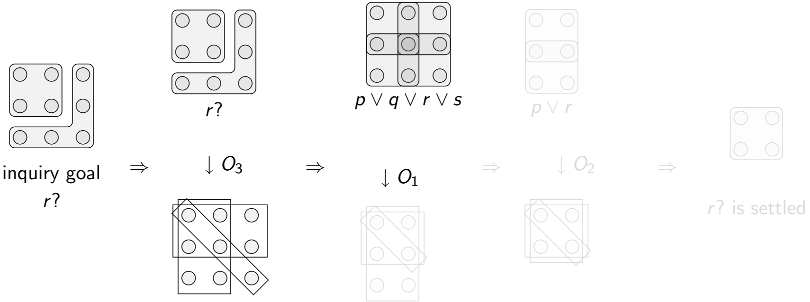

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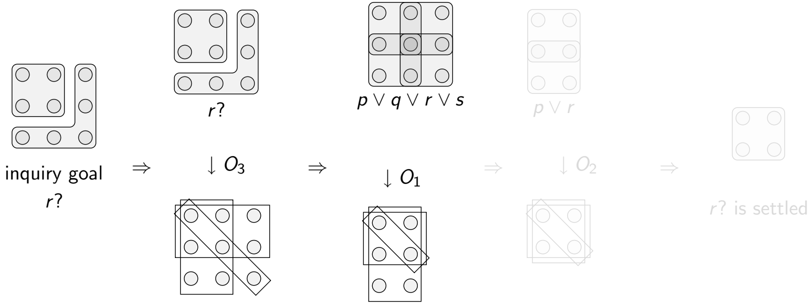

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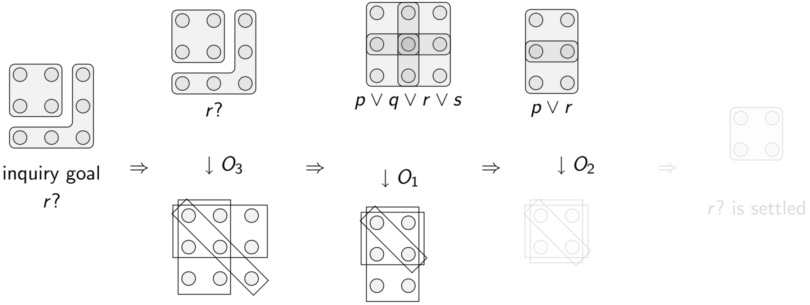

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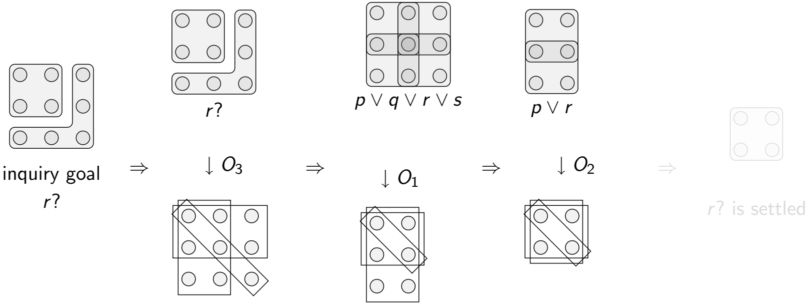

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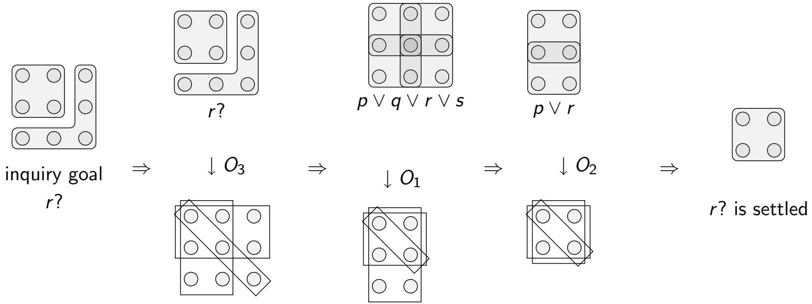

Figure: Interrogative inquiry 1.

Illustrative Example: Inquiry 2

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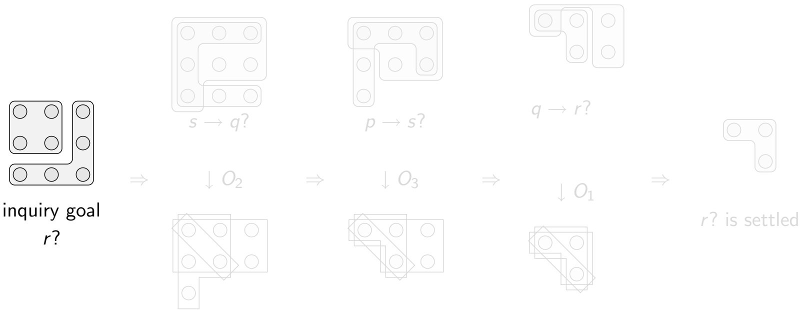

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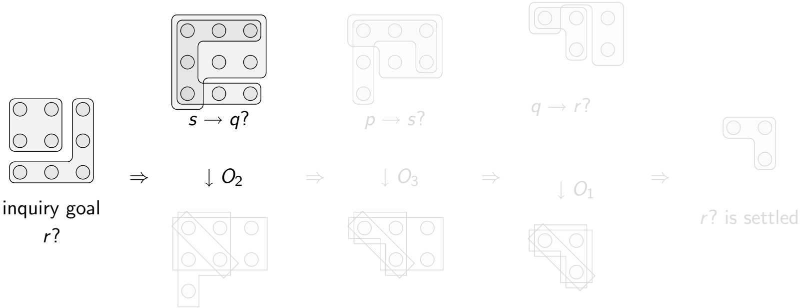

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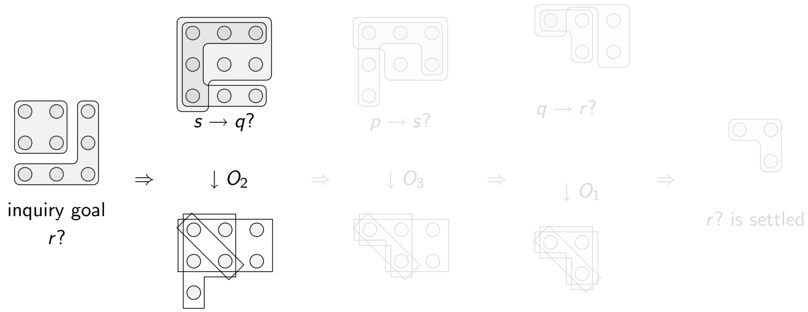

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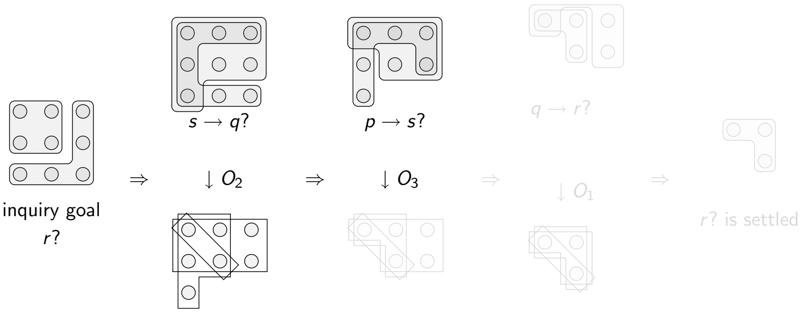

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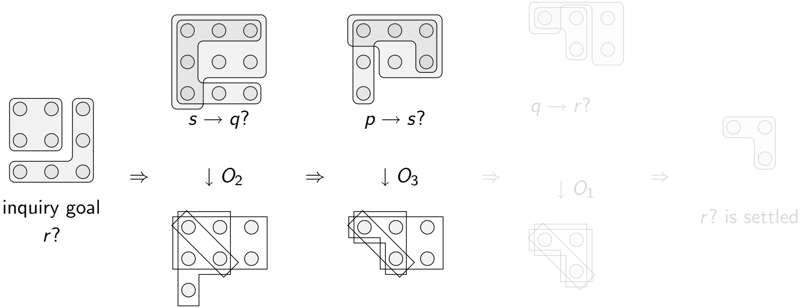

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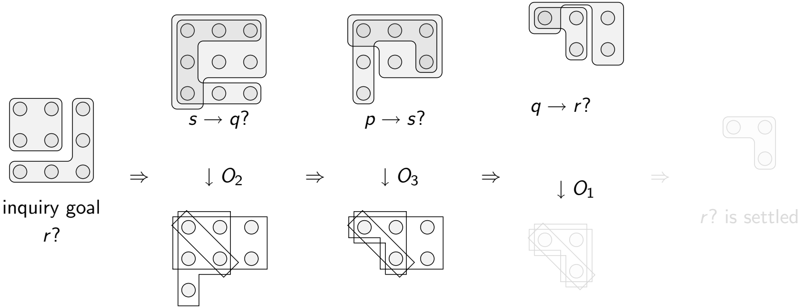

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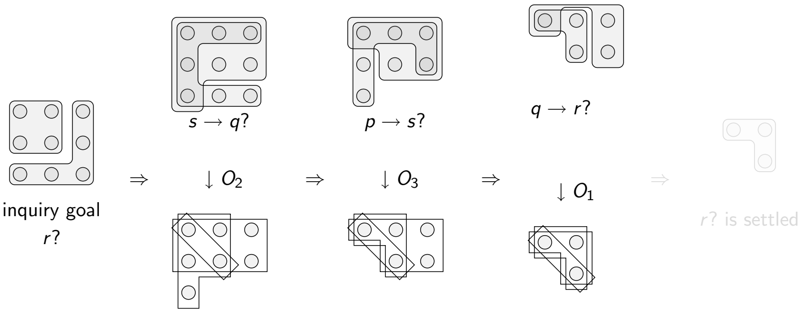

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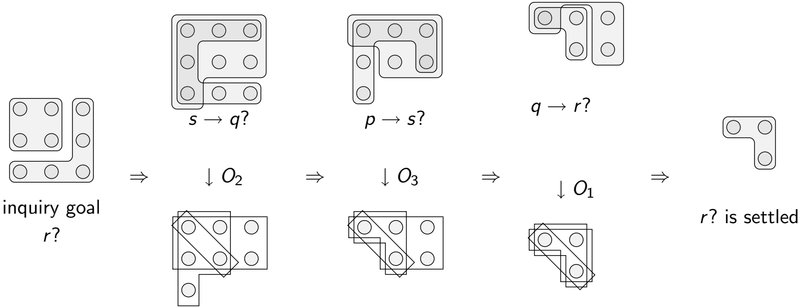

Figure: Interrogative inquiry 2.

Illustrative Example: Inquiry 3

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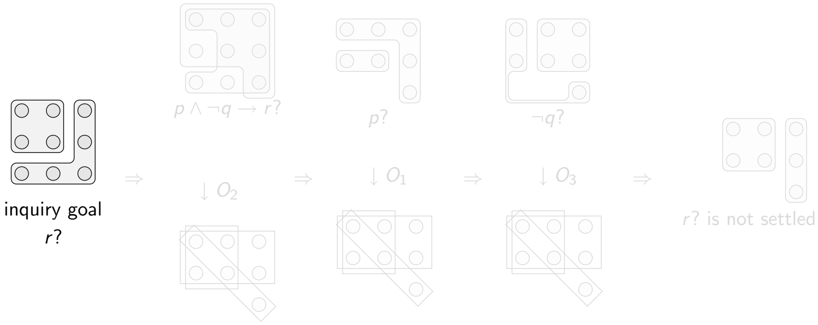
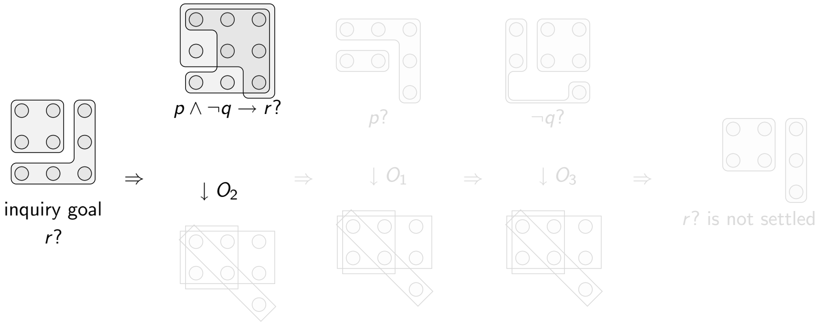


Figure: Interrogative inquiry 3.

Illustrative Example: Inquiry 3



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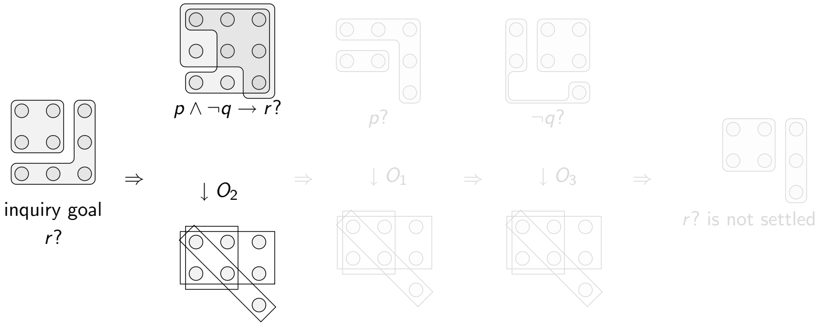

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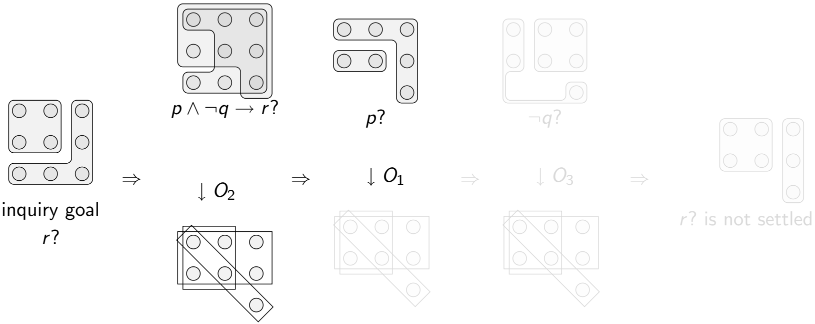

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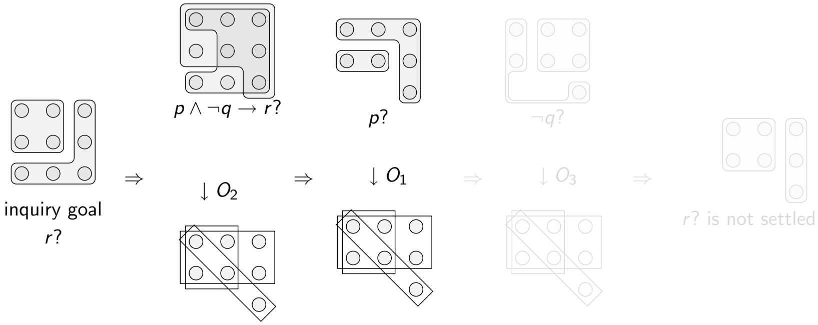

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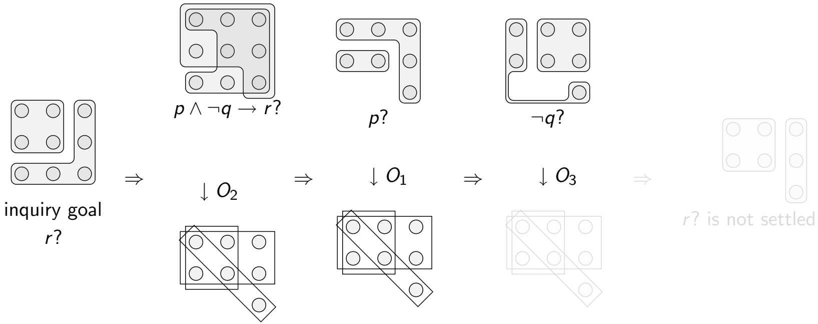

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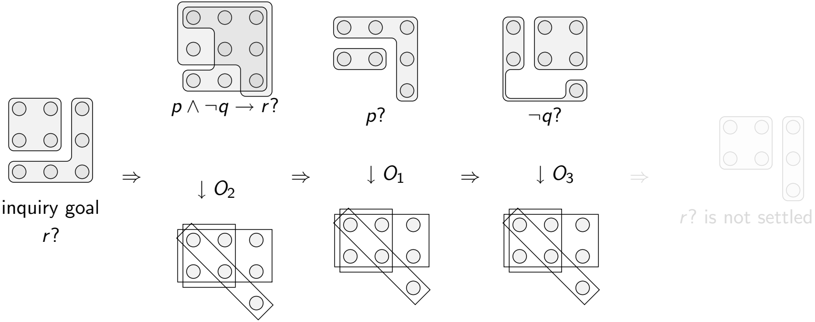

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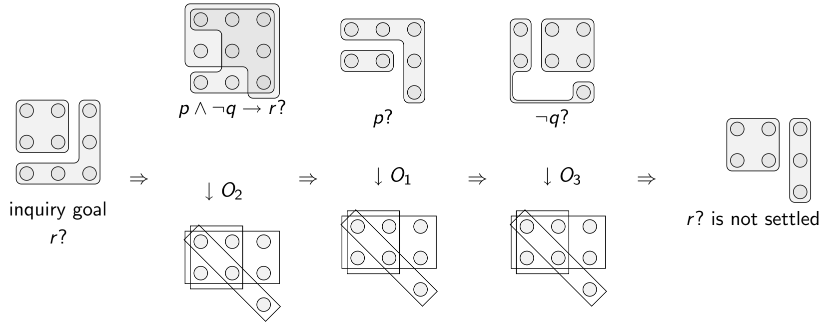

Figure: Interrogative inquiry 3.

Outline

1. The Inquisitive Modelling of Questions and Answers
2. Interrogative Rule
3. Interrogative Protocol, Inquiry and Consequence
4. Logical Aspects
5. Computational Aspects
6. Strategic Aspects of Inquiry: The Algorithmic View

Distributed Information

Distributed Information in Epistemic Logic

φ is distributed information among a group of agents G iff φ is true in all the worlds that every agent in G considers epistemically possible.

Definition (Distributed information)

- *The distributed information state $D(C)$ of C is given by*

$$D(C) := \bigcap_{1 \leq i \leq n} \tau_{O_i} \cap \tau_I,$$

- *The saturated conversational state C_D of C is given by*

$$C_D := (D(C), \ldots, D(C)),$$

- *φ is distributed information in C iff φ is settled in $D(C)$.*

Distributed Information

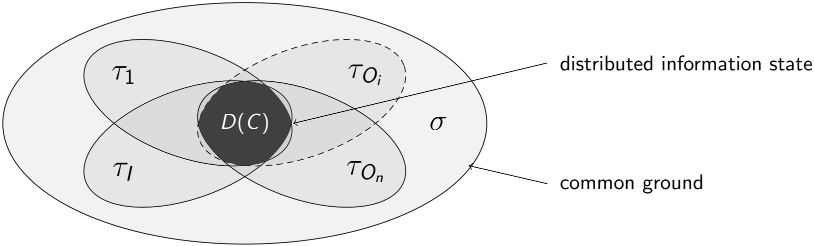

Figure: Common ground and distributed information state.

Theorem (Interrogative Consequence and Distributed Information)

Let $P_?(C, I_n)$ be an interrogative protocol and let $\varphi \in \mathcal{L}$.

φ *is an* **interrogative consequence** *in* $P_?(C, I_n)$

$\Leftrightarrow$

φ *is* **distributed information** *in* C

Proof

$\Rightarrow$. Assume that φ is an interrogative consequence in $P_?(C, I_n)$. By definition, there exists an interrogative inquiry

$$\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k \text{ in } P_?(C, I_n)$$

leading to a node C' such that $\sigma'[\varphi] = \{\sigma'\}$. We show that $D(C) \subseteq \sigma'$.

Proof Continued

Proof Continued (⇒)

Let $v \in D(C)$. Suppose towards a contradiction that $v \notin \sigma'$.

- $v \in \sigma$ so the answer χ_p to φ_p for some $p \in [\![1, k]\!]$ has led to the elimination of v,
- $v \in D(C)$ so v is a member of $\tau_I, \tau_{O_1}, \ldots, \tau_{O_n}$,

$\Rightarrow \chi_p$ must have been eliminative in the informational state τ_{i_p}, not possible by the definition of the notion of answer, so $D(C) \subseteq \sigma'$. Then:

$$D(C) \subseteq \sigma' \text{ and } \sigma'[\varphi] = \{\sigma'\} \quad \Rightarrow \quad D_C[\varphi] = \{\varphi\}$$

Proof Continued (⇐)

Assume $D(C)[\varphi] = \{D(C)\}$. Consider:

$$\langle(\chi_{\tau_I}?, 0), (\chi_{\tau_{O_1}}?, 1), \ldots, (\chi_{\tau_{O_n}}?, n)\rangle_{n+1}.$$

Characteristic Inquiry

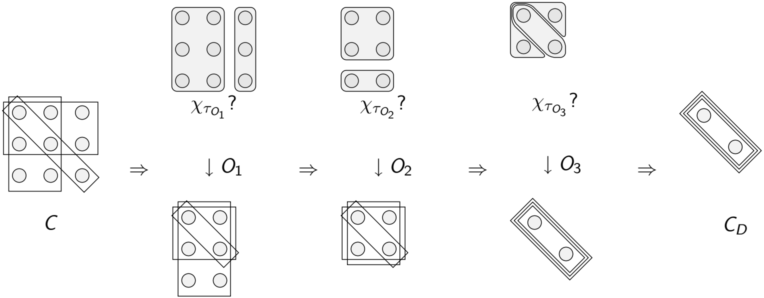

Figure: Characteristic inquiry associated to C from the illustrative example.

The Yes-No Theorem

Theorem (Yes-no Theorem)

Let $P_?(C, I_n)$ be an interrogative protocol and $\varphi \in \mathcal{L}$.

If φ is an **interrogative consequence** *in $P_?(C, I_n)$, then there exists an interrogative inquiry* **composed exclusively of yes-no questions** *which settles φ.*

Proof

Assume φ is an interrogative consequence in $P_?(C, I_n)$. There exists an interrogative inquiry

$\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$ in $P_?(C, I_n)$ leading to C' s.t. $\sigma'[\varphi] = \{\sigma'\}$

Let $\chi_1, \ldots, \chi_k$ be the obtained answers. Then, we claim that

$$\langle(\chi_1?, i_1), \ldots, (\chi_k?, i_k)\rangle_k \text{ leads to } C' \text{ in } P_?(C, I_n).$$

Inquiry 1 from the Illustrative Example

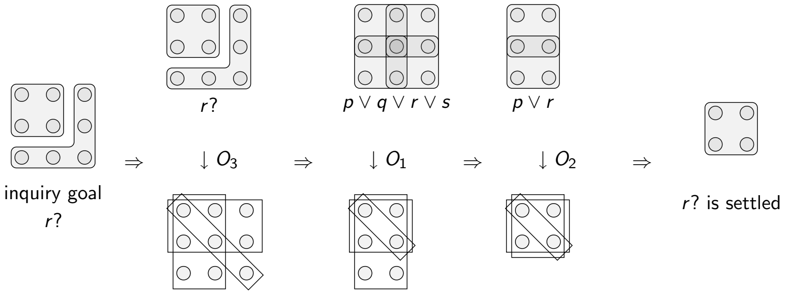

Figure: Interrogative inquiry 1.

Yes-No Inquiry Associated to Inquiry 1

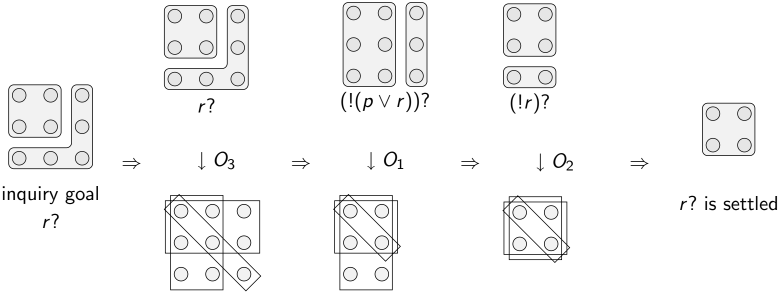

Figure: Yes-no inquiry associated to interrogative inquiry 1.

Outline

1 The Inquisitive Modelling of Questions and Answers

2 Interrogative Rule

3 Interrogative Protocol, Inquiry and Consequence

4 Logical Aspects

5 Computational Aspects

6 Strategic Aspects of Inquiry: The Algorithmic View

Computational Model

- The computational unit is a question-answer step,
- The inquirer has at his disposal his own informational state and the composition of the common ground.

$\Rightarrow$ Focus on **Time Complexity**.

Computational Parameters in $P_?(C, I_n)$

- The number of oracles n,
- The number of indices in inquirer's informational state: $|\tau_I|$,
- The number of indices of the common ground: $|\sigma|$,
- The complexity of the inquiry goal ψ in σ to be settled,
- The pragmatic rules for answering and updating adopted in I_n,
- The cardinality of the set of atomic variables $\mathcal{P}$.

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An Upper Bound to the Process of Interrogative Inquiry

Definition (Scan algorithm)

Let $P_?(C, I_n)$ be an interrogative protocol. For each $v \in \sigma$, where σ refers to the current common ground, the inquirer successively asks the characteristic question χ_v? to each conversational participant.

Proposition

Let $P_?(C, I_n)$ be an interrogative protocol. The output of the scan algorithm is the saturated conversational state C_D.

Theorem (Upper bound)

Let $P_?(C, I_n)$ with $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$. We have:

$$\mathsf{T}_{scan}(C) \leq n \cdot |\tau_I| + 1 \leq n \cdot |\sigma| + 1 \leq n \cdot 2^{|\mathcal{P}|} + 1.$$

The Case of MaxIA

Definition (The all-in-all algorithm)

Let $P_?(C, I_n)$ *be an interrogative protocol. For successively each conversational participant, the inquirer addresses to the considered participant the question*

$$\bigvee_{v \in \sigma} \chi_v$$

where σ *denotes the current common ground and* χ_v *denotes the characteristic proposition of the index* v*.*

Proposition

Let $P_?(C, I_n)$ *be an interrogative protocol such that* I_n *is based on a MaxIA answering rule. The all-in-all algorithm outputs the saturated conversational state* C_D *of* C *in at most* $n+1$ *steps:*

$$\mathrm{T}_{all}(C) \leq n + 1.$$

The all-in-all Algorithm in Action

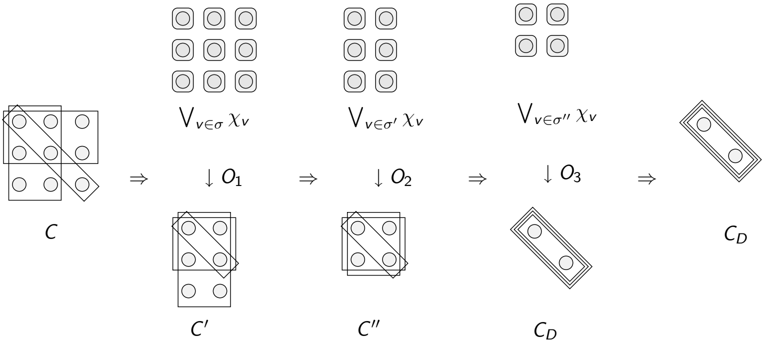

Figure: The all-in-all algorithm applied to the illustrative example.

Interrogative Inquiry and Computational Complexity

Definition (Interrogative inquiry decision problem)

INPUT: *An interrogative protocol* $P_?(C, I_n)$*, a question* φ *in* C *and a natural number* $k \in \mathbb{N}$*.*

QUESTION: *Can* φ *by settled by the process of interrogative inquiry in less than* k *steps?*

Theorem

The Interrogative inquiry decision problem is in **NP***.*

Definition (Interrogative inquiry optimization problem)

INPUT: *An interrogative protocol* $P_?(C, I_n)$*, a question* φ *in* C *and a natural number* $k \in \mathbb{N}$*.*

TASK: *Find an interrogative inquiry settling* φ *which minimizes the number of inquiry steps.*

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Another main requirement that can be addressed to the interrogative approach—and indeed to the theory of any goal-directed activity—is that it must do justice to the strategic aspects of inquiry. Among other things, it ought to be possible to distinguish the definitory rules of the activity in question from its strategic rules. The former spell out what is possible at each stage of the process. The latter express what actions are better and worse for the purpose of reaching the goals of the activity.
[Hintikka, Socratic Epistemology, p. 19]

Two Views on the Strategic Aspects of Inquiry

Definitory and Strategic Rules

- **Definitory Rules:** Tell what are the questions that the inquirer is allowed to ask.
- **Strategic Rules:** Tell what are the best questions to ask in order for the inquirer to reach his inquiry goal.

The Game-Theoretic View

When interrogative inquiry is formalized as a game, the strategic aspects of inquiry can be investigated using the **game-theoretic** notion of strategy.

The Algorithmic View

When interrogative inquiry is formalized through interrogative protocols, the strategic aspects of inquiry can be investigated using the **computational** notion of algorithmic method.

One of the most attractive features of my model of scientific inquiry is that it enables us to use game-theoretical concepts and methods. [...] Game theory is the best available general tool for considerations of strategy. This should make my model especially attractive for the purpose of studying such dynamics of science as are manifested in sequences of choices by a scientist, as distinguished from, e.g., a one-shot choice of a hypothesis on the basis of evidence. Along the same lines, we can also hope to cash in on one of the favorite metaphors of recent theorists of science, the idea of research strategy. The study of research strategies can now in principle be subsumed under the study of strategies in general.
[Hintikka, 'On the logic of an interrogative model of scientific inquiry', p. 81]

Strategic Aspects of Inquiry and Pragmatics

In past studies of various kinds of dialogues, philosophers and linguists have typically formulated their concepts and theses in a way that, in terms of my model, apply to individual moves in the interrogative "game", corresponding to particular utterances in a dialogue or discourse. Examples are provided by speech-act theories, whose very name betrays their conceptual focus; and Grice's conversational maxims. There is a sense in which no such theory focusing on particular "moves" can be fully satisfactory, for from game theory we know that no values ("utilities") can in the last analysis be assigned to individual moves in the game, only to (complete) strategies. In other words, there is no theoretically satisfactory way of relating particular moves to the general ends of the dialogue in question, in the case of my model, to the ends of inquiry.

[Hinikka, 'A spectrum of logics of questioning', p. 137]

Wrapping Up

Interrogative Rule

Characterizes the question-answer steps that the inquirer can make by integrating **pragmatic rules for answering** and **updating**.

Interrogative Protocol

Governs interrogative inquiry as a temporal process and takes as parameters a **conversation state** C and an **interrogative rule** I_n.

Interrogative Inquiry

Refers to a finite sequence of directed questions in a given **interrogative protocol**.

Interrogative Consequence

Refers to the information that can be reached by the process of **interrogative inquiry**.

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First-Order Case

To develop a formalization of interrogative inquiry based on first-order inquisitive semantics.

Interrogative Inquiry about Higher-Order Information

To develop a formalization of interrogative inquiry based on inquisitive dynamic epistemic logic.

Introducing Deduction into the Picture

To represent both questions and inferences in interrogative inquiry.

Computational Approach to Interrogative Inquiry

To investigate the computational aspects of interrogative inquiry.

The Social Dimension of Interrogative Inquiry

To account for the multi-agent dimension of interrogative inquiry.

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