Taking Hintikka Seriously

Interrogative ~~Logic~~ Games as a General Theory of Reasoning

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The Argument in a Nutshell

- Taking Hintikka [HHM99] seriously:
  Interrogative logic as a general theory of reasoning.
  . . . but *logic* (esp. classical) is a poor basis for a **cognitive** theory of reasoning:
  - 'Mainstream' cognitive science:
    Early attempts at logic-based models (e.g. Piaget) are **discredited** since [Was66]; responses by 'mental logic' and 'mental models' theories were inconclusive; **Bayesian models** have become dominant since [OC94].
  - 'State-of-the-art' cognitive theories of semantic abilities:
    logical reasoning (classical or otherwise) supervenes on language interpretation [SvL08], itself **exapted from planning** (esp. strategies in cooperative games) [BG03, SvL08].

The Argument in a Nutshell

- Taking Hintikka [HHM99] seriously (updated):
  Semantic-interrogative games as a general theory of reasoning

The Selection Task (I)
"The Mother of All Reasoning Tasks"

The standard instructions for *Selection Task* (ST) of [Was66, Was68] (in its abstract version) are:

> Below is depicted a set of four cards, of which you can see only the exposed face but not the hidden back. On each card, there is a number on one of its sides and a letter on the other. Also below there is a rule which applies only to the four cards. Your task is to decide which if any of these four cards you must turn in order to decide if the rule is true. Don't turn unnecessary cards. Tick the cards you want to turn.
>
> **Rule** If there is a vowel on one side, then there is an even number on the other side.
> **Cards** $\boxed{\text{A}}$ $\boxed{\text{K}}$ $\boxed{\text{4}}$ $\boxed{\text{7}}$
>
> (From [SvL08, p. 44])

The Selection Task (II)
Assumptions vs. Results

- Neglecting quantification over letters and numbers, Rule simplifies in: *If P, then Q*; and Wason assumes that the **normative selection** is:

$$(A,\cdot),(7,\cdot) \tag{Nor}$$

- But typical results are:

$(A,\cdot)$	$(A,\cdot),(4,\cdot)$	$(A,\cdot),(7,\cdot)$	$(A,\cdot),(4,\cdot),(7,\cdot)$	misc.
35%	45%	5%	7%	8%

- (Nor)-like selection is higher in 'thematic' variants—where the rule is given a less abstract content.

The Selection Task (III): Explanations

- Typically explanations assume that (Nor) is 'the' *context-independent logically competent* answer; but:

  Abstract *context-sensitive* mechanisms override logical competence; and:
  Thematic either: (a) *familiarity* helps to *recover* the 'logical' answer; or: (b) *context-sensitive* mechanisms output the same selection for other reasons.

- E.g. Oaksford & Chater's (Bayesian) 'rational analysis' in [OC94]:

  Abstract Subjects treat cards as sample, and test Rule as inductive hypothesis; the *optimal data selection* matches the empirical results: $(A, \cdot) > (4, \cdot) > (7, \cdot) > (K, \cdot)$;
  Thematic Conditional are in fact *deontic*: subjects' selections reveal *preferences* w.r.t. the rule (esp. exhausting the potential violators).

The Semantic View (I): Motivations

Stenning and Van Lambalgen argue in [SvL01, SvL04, SvL08] that:

- developments in *formal semantics* undermine the assumption of a context-independent logical competence coextensive with the material conditional interpretation;
- empirical evidence shows that within the same category of ST (abstract/thematic), *subjects are not always doing the same thing*—see e.g. tutorial dialogs reported in [SvL01, SvL08];
- subjects must always: (a) recover the experimenter's *intended interpretation*; then: (b) reason from it to a *solution* (and its report);
- selections vary with the *perceived goal* and the corresponding optimal solution; and they can be manipulated by eliciting—or hindering—specific interpretations.

The Semantic View (II): Pros and Cons

Pros The semantic view accounts systematically for co-variance of selections and *semantic content*, in a way that:

- integrates the developments of formal semantics;
- embeds other accounts when relevant—e.g. when the 'cards-as-sample' interpretation is elicited.

Cons The semantic view provides no formal model for:

- *reasoning to an interpretation* before problem-solving within an interpretation;
- the *heuristic value* of semantic interpretation within problem-solving.

General Aims

Our (general) aim is to:

- provide the semantic view with a **unified account** of reasoning **to** and **from** an interpretation.
- give formal models illuminating the **heuristic value of semantics** in problem-solving;
- build the model 'bottom-up,' in agreement with the hypothesis that:

> "Language exapted the planning capacity, both for syntax and for semantics, and in particular discourse interpretation" [SvL08, p. 178]

... and that reasoning about entailment relations supervenes on 'language interpretation' [SvL08, GJ12b].

Plan

1. A formal model of **interrogative learning**.
2. Approximate as far as possible the 'intended interpretation' of ST, with an **interrogative learning ST problem**.
3. Learning methods that **solve/decide the ST problem**, and an undecidability result.
4. Conclude on *prospects for the model*, and directions for further research.

A Formal Model of Interrogative Learning

Definitions (I): Problems

Definition 1. A *problem* is a triple $P = \langle \mathbf{K}, H, Q \rangle$ s.t. :

- $\mathbf{K}$ is set of **states** s.t. any $\kappa \in \mathbf{K}$, is identified by values for a set of relevant parameters;
- H is a set **histories**—sequences of positions at which some value for some parameters are obtained, gradually identifying an underlying state;
- Q is a *partition of states* in $\mathbf{K}$. □

Rem 1.1 $h \in H$ iff there is some *learning method* (LM) addressing P that generates it—H is the view of an omniscient modeler. Constrains on H apply to *all* LM *for* P.

Rem 1.2 P is *finite* iff H is finite; and with *finite horizon* iff every $h \in H$ is of finite length.

Rem 1.3 If h is of length n, h' *extends* h iff $h'|n = h$, where $h'|n$ is the initial segment of length n of h'; $h(n)$ is the item occurring at the nth position of h; and $h \frown e$ is the extension of h with e.

Definitions (II): Interrogative Learning Method

Definition 2. An *interrogative learning method* (LM) for some $P = \langle \mathbf{K}, H, Q \rangle$ is a (partial) function $l : H \longmapsto (\mathbf{A} \cup \mathbf{Q}) \times Q \cup \{?\}$, where:

- $\mathbf{A}$ is a set of (noninterrogative) *actions*;
- $\mathbf{Q}$ is a set of *instrumental questions*;
- '?' indicates suspension between answers in Q;

$l(H)$ is the subset of H generated implementing l in P. □

Rem 2.1 For some $L = \{l_1, \ldots, l_n\}$, $l_1(H) \cup \cdots \cup l_n(H) \subseteq H$ is the **partial representation** of P induced by L.

Rem 2.2 If the source of answer is *strategic*, **Def. 1** and **Def. 2** can be modified to characterize **games** and **interrogative strategies**.

Rem 2.3 A history $h \in H$ is: (a) l-terminal iff l is undefined for extensions of h; and: (b) *maximal* iff it is l-terminal for *every* l.

Rem 2.4 $\mathbf{Q}$ is general enough to capture *selections* of cards to turn in ST.

Special Cases

Example 1. In the *First-Order Paradigm* of [MO98], for some first-order language $\mathcal{L}$:

- $\mathbf{K} = Mod(\Gamma)$ for some $\Gamma \subseteq \mathcal{L}$; parameters are *literals*, and the relevant values *atomic valuations*; and $\mathbf{Q}$ is any non-trivial partition of $Mod(\Gamma)$.
- for any l: (a) $\mathbf{A} = \emptyset$; (b) $\mathbf{Q}$ includes all *yes-or-no* questions $\{\phi, \neg\phi\}$ where ϕ is atomic, and from the vocabulary of Γ; (c) l can *always* generate a *complete diagram* for some $\kappa \in \mathbf{K}$—i.e. Nature *eventually answers all atomic questions*. □

Example 2. *Interrogative-deductive games* from [HHM99, GJ12b]:

- $\mathbf{K} = Mod(\Gamma)$; parameters are *subformulas of elements of* Γ, and the relevant values, valuations satisfying Γ;
- For any l: (a) $\mathbf{A}$ contains 'semantic actions' based on semantic clauses for $\mathcal{L}$, to analyze elements of Γ, and ϕ, in subformulas; (b) $\mathbf{Q}$ contains *yes-or-no* questions, and questions with presuppositions obtained by actions in $\mathbf{A}$; (c) not all answers are available (even in the limit).
- If Nature never answers, the problem is purely epistemic (deductive), and can be modeled as a (generalized) GTS *game*—see [GJ12b]. □

Definitions (III): Solution, Decision

Definition 3. For some problem $P = \langle \mathbf{K}, E, Q\rangle$, and interrogative LM l, and l-terminal $h \in H$:

- *l stabilizes on q in h* at $h(n)$ iff $l(h|n) = \langle X, q\rangle$ for some (possibly empty) $X \subseteq \mathbf{A} \cup \mathbf{Q}$, and: (a) h is of length $m \geq n$, and for any n' s.t. $n \leq n' \leq m$, $l(h|n') = \langle X', q\rangle$, or: (b) h is infinite, and for any $n' > n$, $l(h|n') = \langle X', q\rangle$ (for some possibly empty $X' \subseteq \mathbf{A} \cup \mathbf{Q}$). l outputs the same answer to Q from $h(n)$ until it stops generating h (if it does).
- *l solves P on h at $h(n)$* iff (i) l stabilizes on q at $h(n)$ and: (b) any $\kappa \in \mathbf{K}$ compatible with values of parameters obtained in h is in q; l gets the answer to Q right in h from $h(n)$ on.
- *l decides P on h* iff h is of *finite* length n, and l *solves* P on h at $h(m)$ for some $m \leq n$. l solves P at $h(n)$ and stops doing anything (immediately, or later).
- l solves (decides) P on $\mathbf{K}$—or solves P *simpliciter*—iff: (a) l solves (decides) P on every l-terminal $h \in H$; and: (b) every $\kappa \in \mathbf{K}$ is (partially) characterized in at least one l-terminal $h \in l(H)$ l gets the answer to Q right in all states, without approximation, even under uncertainty about the actual state. □

Solution, Decision (cont'd)

Rem 3.1 Halting on success is a *special case of decision*: l can stabilize and still output actions (e.g. 'control' questions), before it stops (and decides).

Rem 3.2 A LM for P can solve P without deciding it—"no bell rings" when P is solved (when l stabilizes)—see [Kel04].

Rem 3.3 A consequence of **Def. 1–3** is that *any solvable problem with finite horizon is decidable*: 'nonredundancy' constraints on LM (e.g. halting on success, etc.) are especially relevant for such problems.

Reasoning ‘to’ an interpretation

Reasoning to an interpretation of a *set of instructions* can be modeled as follows:

- A **generalized problem** associated with a set I of instructions, is a family **P**$=P_1, \ldots, P_n$ of learning problems that are *possible interpretations of* I, among those a designated problem, P_i is the *actual* problem.
- An **dynamic representation of** P_i for an agent X at $h \in H_{P_i}$, can be characterized with an *awareness function* mapping h to some problem P'—or some subset of **P**, under uncertainty—that captures the agent's representation of P_i a the last position of h—see [HR06, GJ12a, GJ12b].
- An agent's representation can be **partial**, if she considers some P_j with $H_{p_j} \subset H_{P_i}$; **ambiguous**, if she considers *more than one* P_j; or **incorrect**, if P_i and here representation P_j are incomparable.
- In **problem-solving experiments** reasoning to the *intended* interpretation of I is a **coordination problem** between subject and experimenter.

The “Selection Task” as a Learning Problem

Interpretation in ST (I): “Reasoning to” and “Reasoning from”

Focusing on the standard, abstract version of ST, and empirical subjects:

- Reasoning *to* an interpretation is equivalent to:
  - Use the instructions *heuristically* to select a family of LM L, that yield a **representation**;
  - If the representation is ambiguous, find a criterion to **choose the ‘best.’**
- Reasoning *from* an interpretation is equivalent to:
  - Use the instructions to **order by preference** LM that solve the problem (if any); in particular comply with:

    Don't turn unnecessary cards (Nec)
  - **Report** the selection of the ‘best’ LM.

Interpretation in ST (II): Assumptions

Problem 4 (from [SvL08, ch. 3])**.** (a) The 'standard' instructions yield **no single representation of ST**. (b) Even if restricted to 4-card settings, (Nec) is ambiguous, and induces **no unique ranking**.

To make **Prob. 4**, manageable, we make the following (simplifying) assumptions:

Ass. 4.1 We consider a 'generic' problem $\langle \mathbf{K}_{\mathsf{ST}}, H_{\mathsf{ST}}, Q_{\mathsf{ST}} \rangle$ where: (a) every $\kappa \in \mathbf{K}_{\mathsf{ST}}$ is characterized by **four cards**, with A,K,7 and 4 as only possible values; (b) $h \in H_{\mathsf{ST}}$ is generated by **turning cards**; and: (c) $Q_{\mathsf{ST}} = \{\text{Rule}, \neg\text{Rule}\}$, where Rule satisfies a **material conditional**.
Other representations considered are *subsets of* H_{ST}.

Ass. 4.2 We limit to LM that are *patient*—wait for 'Nature's answers' before they asses Q_{ST}—and *credulous*—do not seek justification; i.e. treat the problem as of **pure discovery** [see HHM99]

The ST Problem (I)

Under **Ass. 4.1**, we have:

Definition 5. The *ST problem* is a triple $P_{\mathsf{ST}} = \langle \mathbf{K}_{\mathsf{ST}}, H_{\mathsf{ST}}, Q_{\mathsf{ST}} \rangle$, where:

- For all $\kappa \in \mathbf{K}_{\mathsf{ST}}$, $\kappa = \{(A, x_1), (K, x_2), (4, x_3), (7, x_4)\}$, with $x_1, x_2 \in \{4, 7\}$, and $x_3, x_4 \in \{A, K\}$;
- For all $h \in H_{\mathsf{ST}}$, $h(n) = \{(A, x_1), (K, x_2), (4, x_3), (7, x_4)\}$, with $x_1, x_2 \in \{\cdot, 4, 7\}$ and $x_3, x_4 \in \{\cdot, A, K\}$; and:

  H0 $h|1 = h_0 = \langle \{(A, \cdot), (K, \cdot), (4, \cdot), (7, \cdot)\} \rangle$no back value is initially known;
  H1 if $h(n) = \{(A, x_1), (K, x_2), (4, x_3), (7, x_4)\}$ and $e = \{(A, x_1'), (K, x_2'), (4, x_3'), (7, x_4')\}$ then: $h \frown e \in H_{\mathsf{ST}}$ iff: (a) $x_i \neq x_i'$, for some x_i; and: (b) if $x_i \neq \cdot$, then $x_i' = x_i$. (a) at least one back is revealed at each position after the first; and: (b) once revealed, no value may be forgotten or hidden again.

- $Q_{\mathsf{ST}} = \{\text{Rule}, \neg\text{Rule}\}$, where $\kappa \in \text{Rule}$ iff $x_1 = 4$ and $x_4 = K$.
  Equivalently (given **K**): $\kappa \in \neg\text{Rule}$ iff $x_1 = 7$ or $x_4 = A$. □

We let κ_0 denote the **underlying state** in $\mathbf{K}_{\mathsf{ST}}$ (or 'state of Nature') in any particular instance of P_{ST}.

The ST Problem (II)

Rem 5.1 For each *instance* of P_{ST}, κ_0 **is uniquely identified at each maximal history**—the last position is identical with some $\kappa_i \in \mathbf{K}_{\mathsf{ST}}$ s.t. $\kappa_i = \kappa_0$ (answers are truthful).

Rem 5.2 Identifying κ_0 **up to inclusion in** Rule (or ¬Rule) suffices to assess Q_{ST} (for truth-tracking methods of **Ass. 4.2**).

Rem 5.3 (a) States which differ only w.r.t. unknown back values at $h(n)$, are indiscernible from κ_0 at $h(n)$. (b) Extensions of h_0 'shrink' indiscernibility.

Rem 5.4 Since $\mathbf{A}_{\mathsf{ST}} = \emptyset$, any LM for P_{ST} is a (partial) function $l : H_{\mathsf{ST}} \mapsto \mathbf{Q}_{\mathsf{ST}} \times Q \cup \{?\}$, with (for any l addressing P_{ST}). Without further restrictions:

$$\mathbf{Q}_{\mathsf{ST}} = \{\mathtt{turn}[S] : S \subseteq \{(A,\cdot),(K,\cdot),(4,\cdot),(7,\cdot)\}\} \qquad (\mathbf{Q}_{\mathsf{ST}})$$

.

How to solve P_{ST}?

Observation 6. *A complete (extensional) representation of* $\mathbf{K}_{ST}$ *is not necessary to solve* P_{ST} *if the rule is read as a material conditional.*

Idea of the proof. From **Rem. 5.1–3**, the 'intensional tests' of the *conjunctive property* "$x_1 = 4$ and $x_4 = K$" or the *disjunctive property* "$x_1 = 7$ or $x_4 = A$" are each sufficient to assess membership of **equivalence classes** w.r.t. Q_{ST}. □

'Intensional tests' are cognitively more realistic [see SvL08, p. 178]. In terms of P_{ST}, one has:

Lemma 7. *Selections of* $(A,\cdot)$ *and* $(7,\cdot)$*, in any sequence, or as a 'one-shot' selection, are:* (a) **sufficient** *to solve* any *instance of* P_{ST}*; but:* (b) **not necessary** *to solve* all *instance of* P_{ST}*.*

Idea of the proof. (a) is immediate from the definition of Q_{ST}.
(b) selecting, $(A,\cdot)$ and $(7,\cdot)$ *alone* is sufficient when resp. $(A,7) \in \kappa_0$ and $(7,A) \in \kappa_0$, to determine whether $\kappa_0 \in$ ¬Rule; selecting both is only necessary when $\kappa_0 \in$ Rule. (a) the two-card selection *always* fulfills *both* tests; (b) a shorter selection may sometimes suffice to fulfill the first test. □

What is "unnecessarry"?

Instruction (Nec) is ambiguous, and can be given **narrow scope** or **wide scope**:

Narrow scope = 'history-bounded.'
Heuristic use: obtain, from some history h, a history h^* identical with h save for steps *redundant in* h to identify κ_0 up to inclusion in Rule or $\neg$Rule.

Wide scope = 'cross-history.'
Heuristic use: obtain, from some history h, a history h^* identical with h save for steps *redundant in some* h' (possibly distinct from h) to identify κ_0 up to inclusion in Rule or $\neg$Rule.

Solving the ST problem

The Wason LM (I)

Definition 8. The *Wason* LM is the function l_W defined as follows:

$$l_W(h \frown e) = \begin{cases} \langle \mathtt{turn}[(A,\cdot),(7,\cdot)],?\rangle & \text{if } h = h_0 \text{ and } e = \emptyset \\ \langle \mathtt{turn}[\emptyset], \text{Rule}\rangle & \text{if } h = h_0 \text{ and } e = \{(A,4),(K,\cdot),(4,\cdot),(7,K)\}, \\ \langle \mathtt{turn}[\emptyset], \neg\text{Rule}\rangle & \text{if } h = h_0 \text{ and } e \neq \{(A,4),(K,\cdot),(4,\cdot),(7,K)\}, \\ \text{undefined} & \text{otherwise} \end{cases} \qquad (l_W)$$

Rem 8.1 (l_W) decides both P_{ST} and the "stronger" (more constrained) **'one-shot' ST problem** P^1_{ST} where $\mathbf{K}^1_{\mathsf{ST}} = \mathbf{K}_{\mathsf{ST}}$, Q^1_{ST}=Q_{ST}, but H^1_{ST} is generated by **one-time** selections—i.e. where a LM can only pick a *single element of* $\mathbf{Q}_{ST}$.

Rem 8.2 If a subject's *representation* is *limited to* P^1_{ST} *or* $l_W(H_{ST})$, and because indiscernibility at h_0 is lifted only after a selection, **scopes of** (Nec) **collapse**.

The Wason LM (II)

Rem. 8.1 can be strengthened as follows:

Observation 9. (l_W) *is the only* LM *deciding* P^1_{ST} *under* (Nec).

Idea of the proof. From **Lem. 7**, we know that $\texttt{turn}[(A,\cdot),(7,\cdot)]$ is sufficient to identify κ_0 up to inclusion in Rule or ¬Rule. The restriction to single selections prevents any 'smaller' selection to both track Nature's answers *and* assess Q_{ST^1} on all states. □

An immediate corollary is:

Corollary 10. (l_W) *is the only* LM *deciding* P_{ST} *with a* single *selection without unnecessary cards.*

The Wason LM (III)

Rem 10.1 (l_W) is equivalent to a **uniform strategy in an extensive game with imperfect information**, against Nature: whatever the underlying state—Nature's strategy—is, Inquirer's action is identical.

Rem 10.2 If subjects' representation of P_{ST} is limited to P^1_{ST}, then by **Obs. 9**, $\texttt{turn}[(A,\cdot),(7,\cdot)]$ is the unique solution, and 'ticking' is trivial, aka (Nor) is the **unique possible selection and report** of a solution (and decision).

Rem 10.3 **Uniform strategies may be dominated** depending on: (a) the cost of questions; (b) the probabilities of states (and answers); (c) overall utility of exhaustive answers. . .
. . . but the **costs and probabilities are unconstrained** by the instructions, and there is no unique way to set them.

Nonuniform solutions to $P_{\textsf{ST}}$ (I)

Definition 11. Let l_1 and l_2 be defined as follows:

$$l_1(h \frown e) = \begin{cases} \langle \texttt{turn}[(A,\cdot)], ? \rangle & \text{if } h = h_0 \text{ and } e = \emptyset \\ \langle \texttt{turn}[\emptyset], \neg\text{Rule} \rangle & \text{if } h = h_0 \text{ and } e = e_1 = \langle (A,7),(K,\cdot),(4,\cdot),(7,\cdot) \rangle, \\ \langle \texttt{turn}[(7,\cdot)], ? \rangle & \text{if } h = h_0 \text{ and } e = e_2 = \langle (A,4),(K,\cdot),(4,\cdot),(7,\cdot) \rangle, \\ \langle \texttt{turn}[\emptyset], \neg\text{Rule} \rangle & \text{if } h = h_0 \frown e_2 \text{ and } e = e_{2.1} = \langle (A,4),(K,\cdot),(4,\cdot),(7,A) \rangle, \\ \langle \texttt{turn}[\emptyset], \text{Rule} \rangle & \text{if } h = h_0 \frown e_2 \text{ and } e = e_{2.2} = \langle (A,4),(K,\cdot),(4,\cdot),(7,K) \rangle, \\ \text{undefined} & \text{otherwise} \end{cases} \tag{l_1}$$

$$l_2(h \frown e) = \begin{cases} \langle \texttt{turn}[(7,\cdot)], ? \rangle & \text{if } h = h_0 \text{ and } e = \emptyset \\ \langle \texttt{turn}[\emptyset], \neg\text{Rule} \rangle & \text{if } h = h_0 \text{ and } e = e'_1 = \langle (A,\cdot),(K,\cdot),(4,\cdot),(7,A) \rangle, \\ \langle \texttt{turn}[(A,\cdot)], ? \rangle & \text{if } h = h_0 \text{ and } e = e'_2 = \langle (A,\cdot),(K,\cdot),(4,\cdot),(7,K) \rangle, \\ \langle \texttt{turn}[\emptyset], \neg\text{Rule} \rangle & \text{if } h = h_0 \frown e'_2 \text{ and } e = e'_{2.1} = \langle (A,7),(K,\cdot),(4,\cdot),(7,K) \rangle, \\ \langle \texttt{turn}[\emptyset], \text{Rule} \rangle & \text{if } h = h_0 \frown e'_2 \text{ and } e = e'_{2.2} = \langle (A,4),(K,\cdot),(4,\cdot),(7,K) \rangle, \\ \text{undefined} & \text{otherwise} \end{cases} \tag{l_2}$$

Nonuniform solutions to $P_{\textsf{ST}}$ (II)

If (Nec) is given **narrow scope**, the following holds:

Observation 12. *(a)* (l_W) *is the only* uniform LM *deciding* P_{ST} *without unnecessary cards. (b)* (l_1) *and* (l_2) *are the only* nonuniform LM *deciding* P_{ST} *without unnecessary cards.*

Idea of the proof. (a) Immediate from **Cor. 10** is equivalent, by **Rem. 10.1**.
(b) From **Lem. 7**, we know that $\texttt{turn}[(A,\cdot),(7,\cdot)]$ is sufficient to identify κ_0 up to inclusion in Rule or $\neg$Rule. Using (Nec) (with narrow scope) heuristically, one cannot eliminate more redundant moves than those eliminated in (l_1) and (l_2). □

Nonuniform solutions to P_{ST} (III)

Rem 12.1 If a subjects representation of P_{ST} is 'rich enough' to include either $l_1(H_{\mathsf{ST}})$ or $l_2(H_{\mathsf{ST}})$, some reason to **prefer uniform strategies** is needed to favor (l_W), or some reason to **exhaust potential counterinstances** (as in some 'deontic' cases).

Rem 12.2 If a subject selects (l_1) or (l_2), she will **report** (Nor) only if she has some reason to report **the 'longest' history**.

Rem 12.3 Depending on the cost of questions and probability of states, the cost of (l_1) and (l_2) may be **lower** than the cost of (l_W).

An unsolvability result

If (Nec) is given **wide scope**, then:

Observation 13. *There is no* LM *deciding* P_{ST} *without unnecessary cards.*

Idea of the proof. Immediate from **Lem. 7.b**, using (Nec) with wide scope heuristically. □

A Coordination Problem

Claim 1. (a) P^1_{ST} is the most likely candidate for an **intended interpretation** of ST; but: (b) the task is **rigged against coordination**.

Support. For (a): **Obs. 9**, and its consequences—no ambiguity for (Nec), trivial report.
For (b): **Prob. 4**, and:

- the interpretation of ST as P^1_{ST} requires additional hypotheses from subjects, even under 4-card reading, and **preference patterns that are not constrained by instructions**.
- Sequential tasks where subjects *play runs* rather than *plan strategies*, seem to be evidence that they discover (l_1) and (l_2) while playing [see SvL08, p. 105-106, 111-112], . . .
. . . but this happens when the computational cost of planning ahead is offset, and the **underlying game is competitive** (unbeknown to subject)—[SvL08] acknowledge the first, but fail to notice the second.

□

Do Deontic Aspects Matter?

Claim 2. The 'deontic' aspect of the rule in thematic tasks is second to the incentive to implement **exhaustive tests or uniform strategies**, in elicitation of (Nor)-like selections.

Support. Preferences (and utilities) are important, but:

- 'Deontic rules' facilitate **exhaustive tests and uniform strategies** when a high utility is assigned to find possible violators/exceptions [see OC94];
- Yet similar preferences can be induced in **non-deontic rules**, by *relevance effects* [see GKSvdH01].

□

One last claim (I)

Our last claim is better expressed *after* arguments that support it.

- **Cognitive needs of strategic planning** in cooperative settings have lead to the evolution of **complex semantic representations** needed for successful reasoning to solutions [BG03].
- Semantic heuristics influence **algorithmic strategies** that can **generate a partial representation of an underlying game sufficient to solve it** without fully extensional treatment (complete strategies) [HR06, GJ12a, GJ12b].
- Games where '**knowledge manipulators**' affect outcome by manipulating player's representation of the game are a good model of (*inter alia*) psychological experiments [PTW11, GJ12a].
- Such manipulations subsume *Bayesian rational analysis* and *Relevance theory*, but in agreement with the evidence of **'default' semantic reasoning** revealed by tutorial dialogs [SvL01, SvL08], while offering a unifying formal model.

One last claim (II)

Claim 3. Algorithmic game theory is the natural formal model to study *problem-solving*, and a useful tool interpret and design experiments.

Support. All the arguments advanced in this talk. . .

. . . and (hopefully) future empirical tests! □

THANK YOU!

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